transformations we obtain the coefficient with (q^{-1}) in the Laurent expansion of the integrand (23) at the point q = 0, which is also the residue of the integrand at this point:

$$a_{-1} = -F' \sum_{n=0}^{\infty} \left(\frac{A}{d}\right)^{n+1} \frac{e_n(\sigma d)}{(n+1)!}, \tag{24}$$

where $e_n(\sigma d)$ is a truncated exponential series,

$$e_n(\sigma d) = \sum_{k=0}^{n} \frac{(\sigma d)^k}{k!}.$$

Then the final solution for the temperature of a liquid moving in an underground channel is written in the form

$$\Theta = \exp\left(-2.76\frac{\xi}{\mathrm{Bi}_Q}\right)\left[\exp\left(2.385\frac{\xi}{\mathrm{Bi}_Q}\right) - \exp(-1.28\mathrm{Fo}') \sum_{n=0}^{\infty}\left(2.385\frac{\xi}{\mathrm{Bi}_Q}\right)^{n+1} \frac{e_n(1.28\,\mathrm{Fo}')}{(n+1)!}\right], \tag{25}$$

where $\mathrm{Fo}' = a\tau'/R^2$.

Equation (25) is an approximate solution of the problem examined here. However, it is considerably simpler than the solution obtained by Van-Heerden.

The graphical comparison of experimental data and the results of calculations with Eq. (25) and the equations of Van-Heerden [1] in Fig. 1 shows that they agree well.

NOTATION

λ_{gr}, thermal conductivity of the ground; c, specific heat; ρ, density; a, diffusivity; Q, unit quantity of heat transferred by the liquid through the channel cross section per unit of time; r, distance over radius from channel axis; $t_{gr}(r, x, \tau)$, temperature of the ground as a function of the radial and axial position and time; $t_a(x, \tau)$, temperature of the liquid; V, linear velocity of the liquid; t_0, initial temperature.

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DISTRIBUTION OF THE DISPERSE FRACTION OF AN INJECTED POLYDISPERSE JET IN A GAS FLOW

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The distribution of a polydisperse droplet jet over a gas flow is theoretically investigated. Results are given for specific nozzles.

Questions of the distribution of a disperse condensed phase over a gas flow are of importance in a whole series of processes of chemical engineering, the cooling of hot gases, combustion, etc. As a rule, this phase is introduced in individual regions using a dispers-

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ing device placed at the inlet cross section and directed either along the flow or at an angle to it. The nozzles usually used for spraying liquids are characterized both by a sufficiently broad spectrum of dimensions of the resulting droplets and by a sufficiently large aperture angle of the spray. This is usually the case for the widely used centrifugal nozzles, where the droplets are distributed uniformly over the cross section of the spray produced — mainly along the generatrix of the spray cone. For straight-jet nozzles, this effect is considerably more weakly expressed; the drop distribution over the spray cross section is sufficiently uniform and its aperture angle is small.

When gradually entrained by the gas flow, drops of different dimensions, with different initial velocities, describe significantly different trajectories. The investigation of the disperse-phase distribution over the flow cross section as a function of the distance requires a careful consideration of this picture as a whole. These difficulties are the reason why the literature includes very few (see [1-3], for example) analytical investigations of the disperse-phase distribution in a flow. This is the case not only for polydisperse sets of drops but even for the three-dimensional motion of individual drops. The present work attempts a theoretical description both of the motion of individual drops and of the distribution of their polydisperse combinations in a flow in conditions of nonuniform disperse-phase injection.

Taking account of the character of these problems, little attention is given below to the analysis of particular characteristics of systems of disperse-phase injection, which has been subjected to fairly detailed study in the existing literature, but instead interest is confined to the processes of disperse-phase propagation in the flow. All the parameters required for this investigation — relating to the disperse composition, velocities, and so on, of the polydisperse combination of drops (particles) introduced in the flow — are assumed to be known from the characteristics of the corresponding nozzles (see [1, 4], for example). The injection of a disperse nonvolative phase in the form of a jet (direct-flow nozzles) or cone with the longitudinal axis either in or perpendicular to the motion of the flow is considered. The influence of the droplet jet on the gas flow is neglected, and its velocity v_G is regarded as a constant. In addition, continuous conditions of nozzle operation are assumed, leading to a steady distribution of the droplets in the flow. Finally, in considering drop motion, all the forces acting on the drops from the gas flow, except for hydraulic drag, will be ignored. This assumption holds if the gravitational displacement of the drops may be neglected in comparison with its displacement by the flow, as is the case in most processes.

The problem is considered in the following formulation. Suppose that a nozzle is placed in the gas flow at some point which is taken as the coordinate origin and injects into the flow a polydisperse droplet jet characterized by the density of the mass distribution function of drops over the diameter and over their angle of ejection with respect to the nozzle axis $f_m(\delta, \theta)$. Then the mass of drops ejected in unit time in the range of angles from θ to $\theta + d\theta$ with respect to the nozzle axis and having diameters of from δ to $\delta + d\delta$ is defined as

$$dm_N = Mf_m(\delta, \theta)\, d\delta d\theta, \quad (1)$$

where M is the total mass flow rate of disperse phase.

The introduction of $f_m(\delta, \theta)$ assumes cylindrical symmetry of the jet emitted by the nozzle with respect to the nozzle axis. Below, attention focuses mainly on the determination of the density of the mass distribution function of the drops with respect to the flow $g_m(\vec{r}, \delta)$, where $\vec{r}$ is the radius vector of the point at which the distribution function is determined. This is called the point of observation. The mass of drops with diameter from δ to $\delta + d\delta$ in the element of volume dV including the observation point $\vec{r}$ will then be specified by the expression

$$dm_G = g_m(\vec{r}, \delta)\, d\delta dV. \quad (2)$$

It is obvious that the function $g_m(\vec{r}, \delta)$ is the most general characteristic of the distribution of disperse phase in the flow. In particular, if it is required to know the total drop mass in volume dV at the point of observation, for example, it is sufficient to integrate Eq. (2) over all drop diameters present in dV. So as to be specific, consider, first of all, a nozzle with its axis perpendicular to the gas flow. Below, the distribution function obtained in this case will be generalized to the case of a parallel nozzle axis.

A plane is drawn parallel to the gas flow so as to pass through the point of observation and the point of emission of the polydisperse jet flow. This plane is called the observation plane. Suppose that it forms an angle θ with the nozzle axis (Fig. 1). The abscissa X is

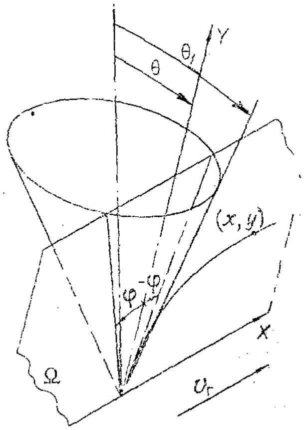

Fig. 1. Initial coordinate system: Ω, observation plane; (x, y), observation point, at which the distribution function is determined.

chosen so as to lie along the flow in this plane, while the Y axis is the projection of the nozzle axis onto the observation plane.

All the drops with a velocity vector lying initially in the observation plane remain there subsequently, since the drag force acting on the drop is directed along the line of drop relative velocity. Suppose that the equation of the trajectory of a drop of diameter δ takes the form

$$F(x,\ y,\ \delta,\ \varphi)=0, \tag{3}$$

where φ is the angle at which the drop is emitted into the gas flow with respect to the axis Y. Specific forms of this dependence will be established below.

Choosing the element of volume at the observation point $dV = yd\theta dydx$, a drop of diameter δ will be in this volume for a time $d\tau = dx/v_x$ where v_x is the component of the drop velocity along the axis X. In this time, for an observation time at an angle of φ to the nozzle, the mass of drops with their diameter in the range from δ to δ + dδ introduced is

$$dm_{\mathrm{N}}=\frac{Mf_m(\delta,\ \theta_1)\cos\varphi}{2\pi\sqrt{1-\cos^2\varphi\cos^2\theta}}\,d\delta d\theta d\varphi d\tau, \tag{4}$$

where $\cos\theta_1=\cos\varphi\cos\theta$ (Fig. 1). This is precisely the mass present in the chosen element of volume at the observation point, since this is a steady problem, and there is no loss of mass. Note that, using the relation between φ and y at fixed δ and x in Eq. (3), $d\varphi$ in Eq. (4) may be replaced by the expression $d\varphi=-dy\dfrac{\partial F}{\partial y}\Big/\dfrac{\partial F}{\partial\varphi}$. Comparison of Eqs. (2) and (4) leads finally to the following expression for the density of the mass distribution of drops over the flow

$$g_m(x,\ y,\ \delta)=-\frac{Mf_m(\delta,\ \theta_1)\cos\varphi}{2\pi v_x y\sqrt{1-\cos^2\varphi\cos^2\theta}}\ \frac{\partial F/\partial y}{\partial F/\partial\varphi}, \tag{5}$$

where v_x is taken at the observation point.

The right-hand side of Eq. (5) includes the angle φ. To obtain the explicit form of the dependence $g_m(x,\ y,\ \delta)$, it is necessary to take account of the relation of φ with x, y, and δ determined by the trajectory of drop motion in Eq. (5) — after performing the differentiation on the right-hand side, of course. This entails solving Eq. (3) with respect to φ (analytically or numerically), and substituting the result into Eq. (5). Below, on the basis of the specific form of the function F, and analytic dependence is obtained for the specific drop-distribution spectra for various geometries of their introduction in the flow.

In the case when the jet axis is parallel to the gas flow, the drop distribution will be of cylindrical symmetry and the symmetry axis will coincide with the jet axis. It is then sufficient to determine the drop distribution in any plane containing the jet axis. Setting

$\theta = 0$ in Eq. (5) and assuming that Eq. (3) is specified in this plane, the abscissa in this plane will then be directed along the flow and coincide with the jet axis, and the ordinate will pass through the point of drop injection. Finally, when the drop in introduced through a jet placed along the flow, Eq. (5) takes the form

$$g_m(x,\ y,\ \delta) = -\frac{Mf_m(\delta,\ \varphi)}{2\pi v_x x}\operatorname{ctg}\varphi\,\frac{\partial F/\partial y}{\partial F/\partial \varphi}, \tag{6}$$

here v_x is taken at the point of observation (x, y), and φ is obtained from Eq. (3).

Simplification of Eqs. (5) and (6) occurs for specific types of nozzle. Thus, in the case of a flow-through nozzle, the angular dependence of the density of the distribution function $f_m(\delta,\ \theta)$ may be neglected and the velocities of all the drops may be regarded as initially parallel. Then the drops in the gas flow will be distributed in a layer of thickness D, where D is the nozzle diameter. Note that, for a flow-through nozzle, it is natural to consider only the case when it is perpendicular to the axis of the gas flow. Neglecting the distribution function on the point of emission of the drops from the sprayer nozzle, it may be assumed that the drop distribution in the plane parallel to the gas flow and including the nozzle axis will also be the true distribution of drops in the gas. The equation of the drop trajectory in this plane — Eq. (3) takes the form (with $\varphi = \pi/2$ = const)

$$F(x,\ y,\ \delta) = 0. \tag{7}$$

It is evident from Eq. (7) that specifying the observation point at once fixes the diameter of the incident drops. Thus, the distribution function in this case depends only on the coordinates of the observation point and expresses the total drop mass in unit volume around this point; Eq. (5) takes the form

$$g_m(x,\ y,\ \delta) = -\frac{Mf_m(\delta)}{Dv_x}\,\delta_D(\delta - \delta^*)\,\frac{\partial F/\partial y}{\partial F/\partial \delta}, \tag{8}$$

where D is the nozzle diameter, and δ^* is the solution of Eq. (7) for δ. Note that the introduction here of δ_D, the Dirac function $\delta_D(z)$, is a formal measure expressing the above-noted fact that only drops of given diameter pass through the observation point in this case. In determining any mass characteristic, including the distribution of the disperse-phase mass over the cross section, it simplifies the integration with respect to δ.

Now consider the case of liquid dispersion using a centrifugal sprayer. Assume that the velocity vectors of the drops emitted by this nozzle lie initially on the generatrix of a cone of root angle 2α. Suppose that the nozzle axis is perpendicular to the gas flow.[1-4] Then the plane passing through the observation point intersects the emission cone along the two generatrices making angles of $\pm\varphi^*$ with the axis Y, where φ^* is given by the expression

$$\cos\varphi^* = \cos\alpha/\cos\theta. \tag{9}$$

As in the case of a direct-flow nozzle, drops of only one diameter pass the point of observation for each of the angles $\pm\varphi^*$. Then the distribution density in Eq. (5) takes the form (for an orthogonal centrifugal nozzle)

$$g_m(x,\ y,\ \delta) = -\frac{Mf_m(\delta)\cos\alpha}{2\pi v_x y\cos\theta\sqrt{\cos^2\theta - \cos^2\alpha}}\,\frac{\partial F/\partial y}{\partial F/\partial \delta}\,[\delta_D(\delta - \delta_1^*) + \delta_D(\delta - \delta_2^*)], \tag{10}$$

where δ_1^* is the solution of Eq. (3) when $\varphi = \varphi^*$, and δ_2^* is the solution when $\varphi = -\varphi^*$. In the case when the axis of the centrifugal nozzle is parallel to the gas flow, drops of only one diameter pass the observation point and in this case Eq. (10) takes the form

$$g_m(x,\ y,\ \delta) = -\frac{Mf_m(\delta)}{2\pi v_x x}\operatorname{ctg}\alpha\,\frac{\partial F/\partial y}{\partial F/\partial \delta}\,\delta_D(\delta - \delta^*). \tag{11}$$

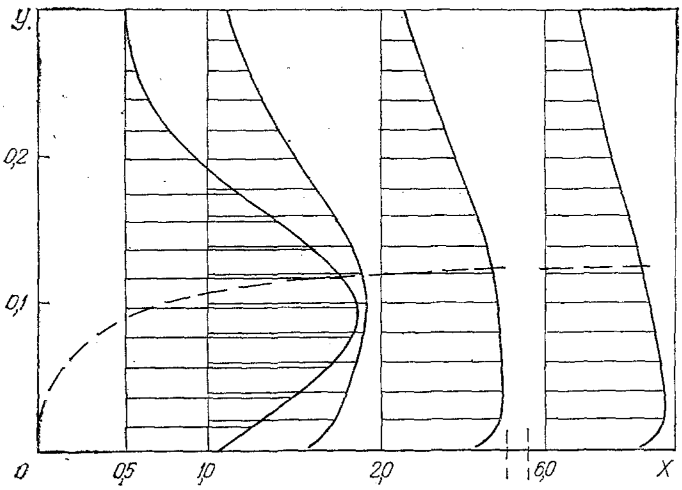

Fig. 3. Profile of the mass distribution of drops Ψ_m for various cross sections of the flow produced by a direct-flow nozzle. x, y, m.

$$y = -\frac{\sin\theta'}{0.81}\tilde{\rho}\delta\left[0.3\left(\mathrm{Re}^{1/2} - \mathrm{Re}_0^{1/2}\right) - \ln\frac{1+0.3\,\mathrm{Re}^{1/2}}{1+0.3\,\mathrm{Re}_0^{1/2}}\right], \tag{16}$$

$$x = y\operatorname{ctg}\theta' - \frac{\delta}{9}\tilde{\rho}\,\mathrm{Re}_G\ln\frac{\mathrm{Re}^{1/2}\left(1+0.3\,\mathrm{Re}_0^{1/2}\right)}{\mathrm{Re}_0^{1/2}\left(1+0.3\,\mathrm{Re}^{1/2}\right)}, \tag{17}$$

where $\sin\theta'$ and $\cos\theta'$ are determined in accordance with Eq. (15), and $\tilde{\rho} \equiv \rho_D/\rho_G$; $\mathrm{Re} = \delta w/\nu$; $\mathrm{Re}_0 = \delta w_0/\nu$; $\mathrm{Re}_G = \delta v_G/\nu$.

Solving Eq. (17) with respect to Re and substituting it into Eq. (16), an equation for the trajectory of the form in Eq. (3) may now be obtained.

The dependence found in this way includes the quantities v_0 and ψ, which have not yet been expressed in terms of the initial parameters. To determine v_0, it is assumed that the nozzle sprays the drops so that, at the moment of ejection, all the drops have the same projection of the velocity onto its axis, equal to V_0. It is simple to find V_0 from the mass flow rate of the nozzle M. For example, for a flow-through nozzle of diameter D, V_0 takes the form $V_0 = 4M/(\pi D^2\rho_D)$. Thus, in the case when the nozzle is directed perpendicular to the flow (Fig. 1), $\psi = \frac{\pi}{2} + \varphi$, and v_0 is easily expressed in terms of its projection V onto the axis Y: $v_0 = V/\cos\varphi$. It follows from geometric conditions that $V = V_0/\cos\theta$, and thus, for the case of a nozzle orthogonal to the flow, the final expression is obtained for v_0 in terms of the flow rate V_0 in the form $v_0 = V_0/(\cos\varphi\cos\theta)$. If the nozzle axis is parallel to the gas flow, however, and directed along the flow, then $\psi = \varphi$ and, if it is opposed to the flow, then $\psi = \pi - \varphi$. In addition, in both cases, $v_0 = V_0/\cos\varphi$. Thus, the parameters v_0 and ψ may be expressed in terms of the angle φ and the basic parameters of the problem. This allows v_0 and ψ to be regarded below as known functions of these quantities, and a final form of the dependence to be obtained for trajectories of the form in Eq. (3).

The solution of the problem obtained gives the distribution function of drops in the flow, but is very cumbersome. To obtain simpler expressions, expedient for engineering calculations, note that $z - \ln(1 + z) \approx z^2/[1.1\,(2 + z)]$ in the range $0 < z \leqslant 8$. Then Eq. (16) is easily solved for $\mathrm{Re}^{1/2}$:

$$\mathrm{Re}^{1/2} = \frac{1.35\,y_A}{\sin\theta'\tilde{\rho}\delta}\left\{\xi + \sqrt{\xi\left[\left(1+\frac{4}{0.3\,\mathrm{Re}_0^{1/2}}\right)^2 + \xi - 1\right]}\right\}, \tag{18}$$

where $= 1 - y/y_A$ and y_A is the asymptotic value of y as $x \to \infty$

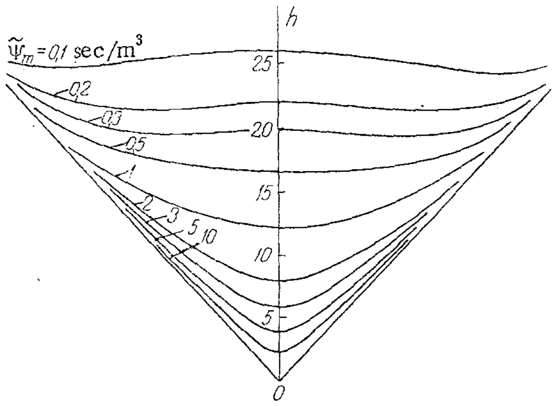

Fig. 2. Form of the drop distribution over the cross section of the flow produced by a centrifugal nozzle. h, cm.

In Eq. (11), δ^* must be taken in the form of the solution for δ from Eq. (3) with Φ replaced by $\varphi^* = \alpha$ or by $\varphi^* = \pi - \alpha$, depending on whether the nozzle is directed along or opposite to the flow.

The dependences obtained in this way – Eqs. (5)-(11) – give the solution of the present problem for various conditions of drop injection in the flow, except for the form of the function $F(x, y, \delta, \varphi)$ from Eq. (3) for the drop trajectory.

The next step is to obtain Eq. (3), which allows specific calculations to be performed by the formula given above. At the same time, taking the requirements of engineering practice into account, the (often very cumbersome) expressions obtained here will tend to be simplified as much as possible. To this end, a series of approximations are made, always stipulating their accuracy.

The equation defining the trajectory of drop motion in the observation plane under the action of a drag force takes the form

$$v_x \frac{\partial w}{\partial x} + v_y \frac{\partial w}{\partial y} = -\frac{3}{4} \frac{\rho_G}{\rho_D} k_c(\mathrm{Re}) \frac{w^2}{\delta}. \tag{12}$$

Here ρ_D and ρ_G are the drop and gas density, respectively; w is the relative velocity of the drop; v_x and v_y are the velocity components of the drop; $k_c(\mathrm{Re})$ is the drag coefficient, depending on Reynolds number. The most general empirical dependence for the drag coefficient, proposed in [1], takes the form

$$k_c(\mathrm{Re}) = \frac{24}{\mathrm{Re}} (1 + 0.183\,\mathrm{Re}^{1/2} + 0.013\,\mathrm{Re}). \tag{13}$$

In the range $0 \leqslant \mathrm{Re} \leqslant 300$, this dependence is approximated by the expression [5]

$$k_c(\mathrm{Re}) = \frac{24}{\mathrm{Re}} (1 + 0.3\,\mathrm{Re}^{1/2}). \tag{14}$$

The range of Re for the drops is completely adequate for most engineering problems, since the drop usually becomes unstable at large Reynolds numbers and breaks into smaller parts. In addition, at large Reynolds numbers, the drop is intensely decelerated with respect to the flow, and the Re rapidly falls in the short initial section to the moderate values considered.

Suppose that a drop entering the gas flow has an initial velocity v_0 directed at an angle ψ to the flow. Then $v_x = v_G + w\cos\theta'$, $v_y = w\sin\theta'$, where θ' is the angle between the relative velocity of the drop and the flow and

$$\cos\theta' = \frac{v_0\cos\psi - v_G}{w_0}, \quad \sin\theta' = \frac{v_0 \sin\psi}{w_0}, \tag{15}$$

where $w_0 \equiv \sqrt{v_0^2 + v_G^2 - 2v_0 v_G \cos\psi}$ is the value of the relative drop velocity at the beginning of trajectory. Solving Eq. (12) with the drag coefficient in Eq. (14) gives

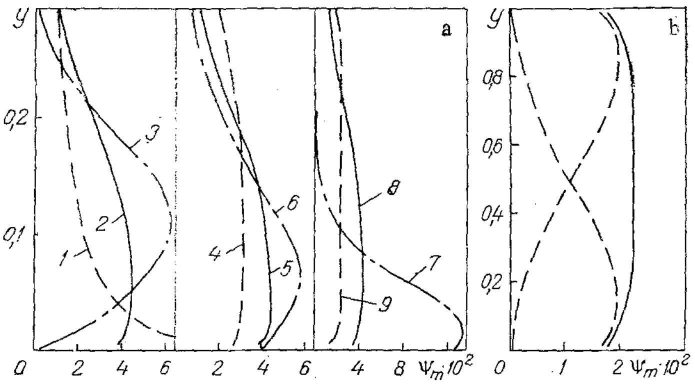

Fig. 4. Dependence of the mass-distribution profile of the drops produced by a direct-flow nozzle on the basic parameters: n = 1 (1), 2 (2), 3 (3); P_G = 1 (4), 5 (5), 10 (6); v_0 = 5 (7), 15 (8), 30 (9) (a); and an example of particle selection for uniformity of the drop distribution over the gas-flow cross section (b).

$$y_A = \frac{v_0 \sin\psi/v_r}{9.9\,(2+0.3\,\mathrm{Re}_0^{1/2})}\tilde{\rho}\delta\,\mathrm{Re}_G. \tag{19}$$

In addition, the logarithm in Eq. (17) may be approximated, after substituting for Re from Eq. (18), by the expression $\Lambda = [0.83 - 0.1\ln(0.18+\mathrm{Re}_0\sqrt{\xi})]\ln\sqrt{\xi}$. Substitution of these approximating relations into Eq. (17) gives a sufficiently accurate description of the trajectories with increasing distance from the point of introduction of the drop in the flow. At the same time, the condition dx/dy = cot ψ must be satisfied at the point of introduction. Imposing this requirement and the usual asymptote-matching procedure, the equation of the trajectory may be finally reduced to the form in Eq. (3) with F in the form

$$F(x,\ y,\ \delta,\ \varphi) = x + \frac{\delta}{9}\tilde{\rho}\,\mathrm{Re}_G\left\{\frac{\ln\xi - \xi + 1}{2}[0.83 - 0.1\ln(0.18 + \mathrm{Re}_0\sqrt{\xi})] - \frac{v_0(1-\xi)\cos\psi}{1.1v_G(2+0.3\,\mathrm{Re}_0^{1/2})}\right\}, \tag{20}$$

which is completely analogous to Eq. (3).

To determine the total error of all the approximations made, an accurate — but very cumbersome — solution of Eq. (12) with the drag coefficient in Eq. (13) is obtained. The results of calculations on the basis of the accurate solution have been compared with Eq. (20) for Re_0 in the range from 0 to 300, and also with variation of the quantity $\gamma = v_0\cos\psi/v_0$. The results of this comparison show that, up to the values $\gamma = \pm 0.6$ and $\mathrm{Re}_0 = 300$, the error in determining the trajectories is known to be no more than 10%. The error may only increase at large γ, which corresponds to the case — encountered very rarely in practice — when the component of the drop injection velocity parallel to the flow is close to, or exceeds, the gas-flow velocity. In usual conditions, however, when γ and Re_0 are smaller, the error which arises rapidly decreases; for smaller Re, the error remains small even for considerably larger γ, i.e., even for the case of high injection velocities.

Thus, it may be assumed that Eq. (20) is a good approximation for the equations of the trajectory, completely adequate for use in Eqs. (5), (6), (8), (10), (11) for g_m. Finally, as regards the error in the very final values of the distribution g_m due to these approximations, taking their integral character into account, it is evident that they should not exceed the given values of the trajectory errors.

In the dependences obtained in this way for g_m, the expression for v_x is still fairly cumbersome. Taking account of Eq. (15), it may be written in the form

$$v_x = v_G + \frac{w}{w_0}(v_0\cos\psi - v_G),$$

where it must be remembered that $w = \mathrm{Re}\nu/\delta$, and Re is determined from Eq. (18). In this connection, it is expedient to obtain a simpler approximate expression for v_x, in the form

$$v_x = v_G \left[1 + \xi \left(\frac{v_0}{v_G} \cos\psi - 1 \right) \frac{1 + 0.08\xi\, \mathrm{Re}_0^{1/2}}{1 + 0.08\, \mathrm{Re}_0^{1/2}} \right]. \quad (21)$$

The relative error of this approximation when $-1 \leqslant \gamma \leqslant 4$ (which is known to be broader than the above-noted range of actual variation of the parameter $\gamma = v_0 \cos \psi / v_G$) does not exceed 1%.

It follows from the form of the function g_m that it does not include the function F but the ratio of its partial derivatives with respect to y and δ. It is obvious that the expression for this ratio may easily be obtained by direct differentiation of Eq. (20) for F. Finally, the expressions for v_0 and ψ in terms of φ appearing in g_m were also given above.

Note, in conclusion, that the influence of possible change in drop size (for example, its evaporation in the hot gas flow) on the mass distribution function is not considered directly in the present work. Nevertheless, in most of the cases encountered in practice, the process of drop evaporation occurs considerably more slowly than its entrainment by the flow. Accordingly, the above consideration may evidently also be used for the analysis of processes of this type. If it proves necessary to take account of evaporation, however, than the distribution function over the gas flow is obtained accurately as in Eq. (5). It is only necessary here to take account additionally of the change in δ in comparison with Eqs. (2) and (4).

As an example of the use of the dependences obtained, some results are now given of the calculation of the distribution of the dispersional attachment in the combustion chamber of a magnetohydrodynamic unit. Graphs of the distribution of drops with respect to the mass in the flow of natural-gas combustion products are given here. Consider the injection of a polydisperse jet of drops (particles) of K_2CO_3 with the following distribution function from [4]

$$f_m(\delta) = n \frac{\delta^{n-1}}{d_0^n} \exp\left\{ -\left(\frac{\delta}{d_0} \right)^n \right\}.$$

The following parameters are taken as the standard set: pressure P_G = 5 bar, temperature T_G = 2800°K, gas velocity v_G = 100 m/sec, median diameter $d_m \equiv d_0 \sqrt[n]{\ln 2} = 100$ μm, n = 2, particle injection velocity V_0 = 15 m/sec.

The particle distribution in a cross section 1 m from the position of a centrifugal nozzle orthogonal to the flow is shown in Fig. 2. Lines of constant $\tilde{\Psi}_m$ are shown, where $g_m = \frac{M}{2\pi} \tilde{\Psi}_m$; see Eq. (10).

The distribution of particles introduced in the flow by an orthogonal direct-flow nozzle in various cross sections is shown in Fig. 3. The dashed curve shows the trajectory of motion of particles with the diameter d_m. Here and below, for direct-flow nozzles $g_m = \frac{M}{D} \Psi_m$; see Eq. (8).

It is evident from Fig. 3 that drops with small diameters rapidly reach the asymptotic sections of the trajectory and form a boundary part of the distribution that is practically unchanging with distance from the nozzle. Larger particles lead to extension of the distribution pattern into the depth of the flow with increase in distance of the cross section from the nozzle.

Variation in various parameters of the flow and the nozzle shows that the distribution of the disperse phase depends weakly on T_G in the range 2000-3200°K, but markedly on P_G, on the initial partial velocity V_0, on the gas velocity v_G, and also on the median drop dimension and on n. In Fig. 4a, the distribution of Ψ_m in a cross section 2 m from the direct-flow nozzle is shown. The parameters indicated in the figure are varied here, but all the other parameters remain "standard."

In a whole series of engineering processes, ensuring sufficient efficiency of their occurrence involves ensuring that the distribution of the disperse phase over the channel cross section be as uniform as possible. The possibility of using the given model in choosing the optimal nozzle position to achieve this uniformity is illustrated in Fig. 4b. The data are shown for a channel cross section 2 m from two diametrally positioned direct-flow nozzles, orthogonal to the flow. The channel diameter is 1 cm, the velocity of disperse-phase injection is the same for both nozzles (35 m/sec); all the other parameters of the nozzles and the gas flow are "standard." The dashed lines in Fig. 4b show the distributions given by each nozzle and the continuous curve shows the total mass distribution function of drops over the channel cross section. It follows from the resulting calculations that the correct choice of parameters of the injection system may ensure a very homogeneous distribution of the disperse phase even with a small number of nozzles.

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PROPAGATION OF SMALL DISTURBANCES IN CONCENTRATED DISPERSED SYSTEMS

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The mechanism of elastic pressure waves in concentrated dispersed systems is discussed. It is shown that the continuously relaxing medium model is valid for describing acoustic effects in a fluidized vibrating layer.

Concentrated dispersed systems of the "fluidized layer" (FL) type are characterized by the essential nonstationarity of all hydrodynamic processes due to the nonlinear properties in the particle bulk concentration. The propagation laws of dynamic disturbances play an important role. The dynamic FL characteristics earlier considered were usually related to propagation of comparatively slow plastic isolated waves during spontaneous or induced change of flow of a fluidized agent [1, 2]. Their appearance was related to quasielastic relaxation processes due to the nonlinear dependence of the aerodynamic particle resistance on their spacing density, earlier found by Roy [3]. In this case the analysis included only low layers, with pressure waves propagating along them practically instantaneously [4, 5]. One of the most interesting effects, explained within the concepts of incompressibility of the fluidized agent in an FL, is the effect of ordered oscillations of the gas pressure and of the dispersed phase density (in the form of self-oscillations in boiling [6] and induced oscillations in pulsating [1] and vibrating-boiling [5] layers), characterizing the law of expansion cycle, precipitation at each period of oscillation, occurring with a completely determined "zero-order" frequency $f_0 \sim \sqrt{g/H_0}$.

Imposing on an FL induced oscillations with a frequency larger than the zero-order frequency ($f_B > f_0$), the action of the relaxation oscillations is restricted by the surface and bottom portions of the layer, and their contribution to the formation of the internal portions of the FL is diminished. At the same time the passage of a pressure wave through a high FL is compatible with an oscillation period T_B. Under these conditions one must expect the appearance of gas compressibility (elasticity), which would lead to resonance effects of higher than "zero" order.

At present the model of interacting, mutually penetrating continua is most widely used to describe the behavior of dispersed systems. This model is valid when the characteristic

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